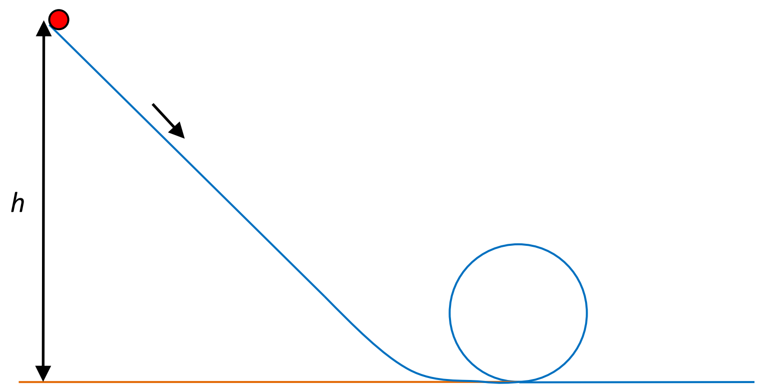
# Teacher notes
# Topic A

## An instructive problem in rotational motion

A point particle of mass $m$ starts from rest at a height $h$ from the ground. It enters a loop-the-loop machine of radius $R$.




(a) Show that the particle does not fall off the track if $h > \frac{5}{2}R$.

The particle is replaced by a marble of mass $m$ and radius $r$. The moment of inertia of the marble is $\frac{2}{5}mr^2$. The marble rolls without slipping.

(b) Show that the marble does not fall off the track if $h > \frac{27}{10}R$.

(c) The marble is released from rest at a height $h = 3R$. The marble enters the loop. Determine the magnitude of the horizontal force acting on the marble when at a height $R$ from the ground.

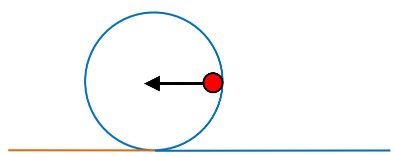
Solution

(a) The speed at the top of the loop is found from $mgh = \frac{1}{2}mv^2 + mg(2R)$ i.e. $v^2 = 2gh - 4gR$. The net force at the top is $N + mg$ and so $N + mg = m\frac{2gh - 4gR}{R}$. This gives

$N = m\dfrac{2gh - 4gR}{R} - mg = m(\dfrac{2gh}{R} - 5g) = mg(\dfrac{2h - 5R}{R})$. The particle will not fall off the loop if $N > 0$ i.e. if $h > \dfrac{5}{2}R$.

(b) Now, $mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}\dfrac{2}{5}mr^2\omega^2 + mg(2R)$ and since $v = \omega r$, $mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}\dfrac{2}{5}mv^2 + mg(2R)$, giving $gh = \dfrac{7}{10}v^2 + g(2R)$, i.e. $v^2 = \dfrac{10gh - 20gR}{7}$. Then $N + mg = m\dfrac{10gh - 20gR}{7R}$ and so $N = m\dfrac{10gh}{7R} - \dfrac{20}{7}mg - mg = mg\left(\dfrac{10h - 27R}{7R}\right)$. The marble will not fall off the loop if $N > 0$ i.e. if $h > \dfrac{27}{10}R$.

(c) Now, $mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}\dfrac{2}{5}mr^2\omega^2 + mgR$ and since $v = \omega r$, $mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}\dfrac{2}{5}mv^2 + mgR$, giving $gh = \dfrac{7}{10}v^2 + gR$, i.e. $v^2 = \dfrac{10gh - 10gR}{7} = \dfrac{30gR - 10gR}{7} = \dfrac{20gR}{7}$.



The horizontal force is just $\dfrac{mv^2}{R} = m\dfrac{\frac{20gR}{7}}{R} = \dfrac{20}{7}mg$.